International Mathematics Assessments for Schools

2021 ~ 2022 JUNIOR DIVISION FIRST ROUND PAPER

Time allowed：75 minutes

When your teacher gives the signal, begin working on the problems.

INSTRUCTION AND INFORMATION

GENERAL

1. Do not open the booklet until told to do so by your teacher.
2. No calculators, slide rules, log tables, math stencils, mobile phones or other calculating aids are permitted. Scribbling paper, graph paper, ruler and compasses are permitted, but are not essential.
3. Diagrams are NOT drawn to scale. They are intended only as aids.
4. There are 20 multiple-choice questions, each with 5 choices. Choose the most reasonable answer. The last 5 questions require whole number answers between 000 and 999 inclusive. The questions generally get harder as you work through the paper. There is no penalty for an incorrect response.
5. This is a mathematics assessment, not a test; do not expect to answer all questions.
6. Read the instructions on the answer sheet carefully. Ensure your name, school name and school year are filled in. It is your responsibility that the Answer Sheet is correctly coded.

THE ANSWER SHEET

1. Use only pencils.
2. Record your answers on the reverse side of the Answer Sheet (not on the question paper) by FULLY filling in the circles which correspond to your choices.
3. Your Answer Sheet will be read by a machine. The machine will see all markings even if they are in the wrong places. So please be careful not to doodle or write anything extra on the Answer Sheet. If you want to change an answer or remove any marks, use a plastic eraser and be sure to remove all marks and smudges.

INTEGRITY OF THE COMPETITION

The IMAS reserves the right to re-examine students before deciding whether to grant official status to their scores.

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Questions 1-10, 3 marks each

1. The equation $n = 456 - 3T$ is used to define the relationship between the number of hot chocolate cups *n* sold per day in a coffee shop and the average daily temperature *T*, in degrees Celsius. According to the model, what does the number "3" mean in the equation?
   (A) For every increase of $3°C$, one more cup of hot chocolate will be sold;
   (B) For every decrease of $3°C$, one more cup of hot chocolate will be sold;
   (C) For every increase of $1°C$, 3 more cups of hot chocolate will be sold;
   (D) For every decrease of $1°C$, 3 more cups of hot chocolate will be sold;
   (E) For every decrease of $1°C$, 3 times of cups of hot chocolate will be sold.

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2. What is the sum of all positive integers *n* that satisfy the inequality $\frac{7}{18} < \frac{n}{5} < \frac{20}{7}$?
   (A) 25　　(B) 56　　(C) 78　　(D) 104　　(E) 105

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3. In the given grid, an ant begins at the "Start" cell and in each move, it can go either one step right or down to any adjacent cell with common side until it reaches the "End" cell. If in each move, it gets the number in the square, then what is the smallest sum the ant can get?

Start 	18	15	13	14
20	21	12	26	21
17	7	11	19	10
6	8	17	24	11
10	20	4	19	**End**

   (A) 91　　(B) 95　　(C) 96　　(D) 97　　(E) 102

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4. A set of 13 positive integers have a mean of 8 and a median of 9. What is the greatest possible integer that is found in this set? (Note: The *mean* of the set is the sum of the all numbers of the set divided by the count of numbers of the set. The *median* of the set is the "middle" number, when the all numbers of the set are listed in order from smallest to greatest.)
   (A) 26　　(B) 32　　(C) 38　　(D) 42　　(E) 44

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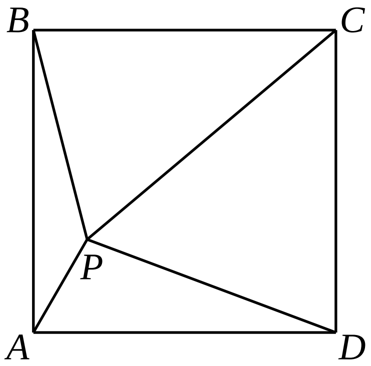
5. In the diagram below, $ABCD$ is a square and P is a point inside it. It is known that $PA = 1\text{cm}$, $PB = 2\text{cm}$ and $PC = 3\text{cm}$, what is the length, in cm, of PD?



（A）$\sqrt{2}+\sqrt{3}$ （B）$\sqrt{6}$ （C） 4 （D）$\sqrt{17}$ （E）6

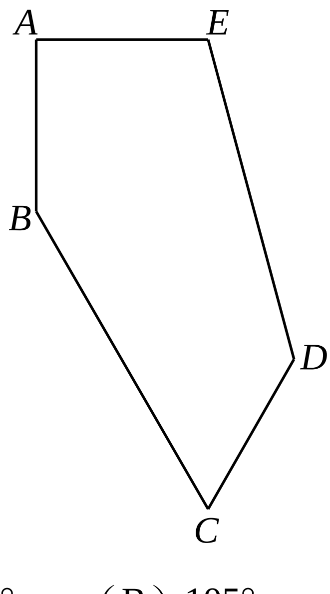
6. Let $\overline{ab}$ be a two-digit number. Now, swap its digits to form another two-digit number, multiply it by 5 and take the remainder of the result when divided by 9. If the resulting number is 5, then how many different $\overline{ab}$ exist?

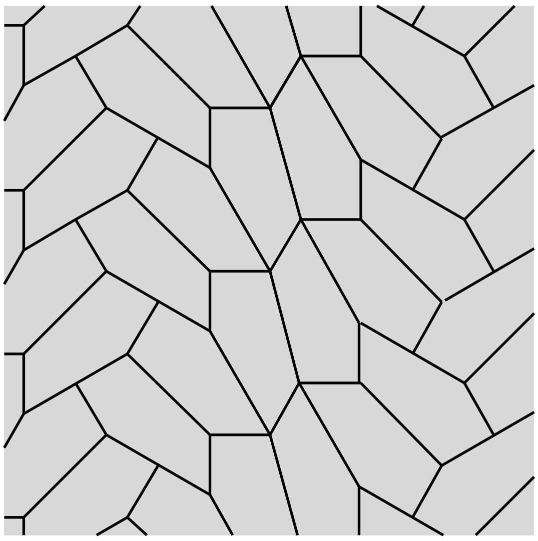
（A）7 （B）8 （C）9 （D）10 （E）11

7. At most how many numbers can you choose from the set {1, 2, …, 10} such that the positive difference of any two chosen numbers is not 4, 5 or 9?

（A）4 （B）5 （C）6 （D）7 （E）8

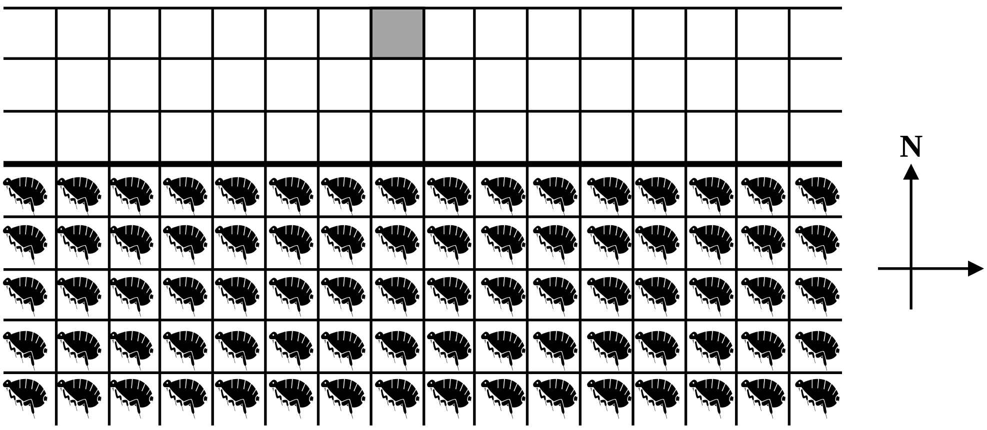
8. As show in the diagram below, we use some number of pentagons, that is identical to $ABCDE$ to fill the plane below. What is the angle measure, in degrees, of $\angle ABC$?





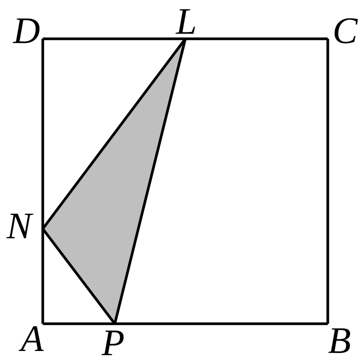
（A）100° （B）105° （C）120° （D）135° （E）150°

9. The world of the beetles consists of one entire plane divided into unit squares. Initially, all the beetles are located at squares south of an inner wall and each square is occupied by at most one beetle. In each move, a beetle can jump over another beetle in an adjacent square and land on the square immediately beyond. The beetle that was jumped over is then removed. However, the move is not permitted if the square is already occupied. The jump may be northward, eastward or westward. At least how many beetles are needed so that one beetle reaches the shaded square in the diagram below?



（A）5 （B）6 （C）7 （D）8 （E）10

10. In the diagram below, *ABCD* is a square with side length 12 cm. Point *P* is on side *AB* such that *AP*:*PB* = 1:3, point *L* is on side *CD* such that $CL = DL$ and point *N* is on side *DA* such that *DN*:*NA* = 2:1. What is the area, in cm^2, of the triangle *PNL*?



（A）24 cm^2 （B）30 cm^2 （C）48 cm^2 （D）90 cm^2 （E）144 cm^2

Questions 11-20, 4 marks each

11. What is the sum of all possible positive integers *a*, such that for any positive integer $n > 100$, $\dfrac{n(n+2)(n+4)}{a}$ is an integer?

（A）1 （B）2 （C）3 （D）4 （E）6

12. The distance between Frankfurt and New York is 6750 km and the time difference between both cities is 6 hours. If it is 08:00 in New York right now, then it is 14:00 already in Frankfurt. At 15:00 Frankfurt time, an Airbus 320 with a cruising speed of 700 km/h departs from Frankfurt to New York. While at 10:00 New York time, a Boeing 787 plane with a cruising speed of 800 km/h departs from New York to Frankfurt. The route goes over the Atlantic Ocean and it is known that there is a constant wind blowing from New York to Frankfurt at a speed of 100 km/h. It is also known that the Airbus 320 cannot fly more than 5 hours without re-fueling, while the Boeing 787 can fly for over 12 hours without re-fuelling. The Airbus 320 will need to re-fuel at a small airport, which is 2400 km from Frankfurt. The re-fuelling and take-off will require 1 hour in total. What will be the time in New York when the two planes meet assuming they fly the same trajectory?
（A）13:30 （B）14:30 （C）15:30 （D）16:30 （E）20:30

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13. What is the remainder when $1^2+2^2+3^2+\dots+2020^2+2021^2$ is divided by 4?
（A）2021 （B）1011 （C）1 （D）2 （E）3

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14. If $a=\sqrt[3]{4}+\sqrt[3]{2}+\sqrt[3]{1}$, then what is the value of $\frac{3}{a}+\frac{3}{a^2}+\frac{1}{a^3}$?
（A）1 （B）7 （C）$\sqrt[3]{6}$ （D）$\frac{19}{8}$ （E）$\sqrt[3]{4}$

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15. Let *a* be a positive integer, where the positive difference of every two digits of *a* in base 10 are all written on a board. If some of the numbers are erased and only the numbers 2, 0, 2 and 2 are left, what is the least possible value of *a*?
（A）1011 （B）1112 （C）1113 （D）2000 （E）2022

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16. There are three integers *x*, *y* and *z* such that $x \le y \le z \le 8$, $x+y+z=12$ and $xy+yz+zx=27$. What is the value of *xyz*?
（A）40 （B）8 （C）0 （D）−8 （E）−40

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17. Charlie and Danny bought some apple and carrot juices in the supermarket. Charlie bought some brand *A* apple juices which costs \$5 per bottle and some brand *B* carrot juices which costs \$1.5 per bottle, spending a total of \$139.5. Meanwhile, Danny bought some brand *C* apple juice which costs \$4.5 per bottle and some brand *D* carrot juices which costs \$2 per bottle, spending a total of \$167. If it is known the number of fruit juices that each of them bought is the same, then how many brand *A* apple juices did Charlie buy?
（A）15 （B）18 （C）21 （D）25 （E）33

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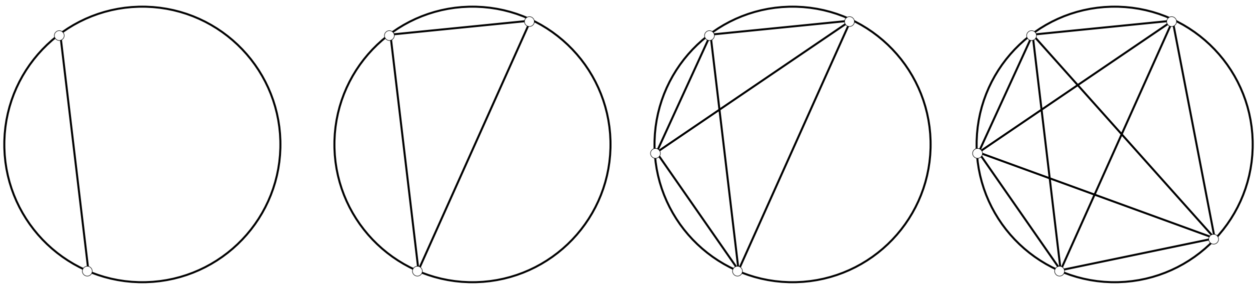
18. It is known that Annie can wash 3 plates every minute, while Betty can wash 2 plates every minute. Also, it is known that Annie can wash 9 cups every minute, while Betty can wash 7 cups every minute. If there are a total of 134 dirty plates and cups and both of them worked together in washing all the dirty plates and cups and finished in exactly 20 minutes, how many dirty plates are there?
(A) 42 (B) 49 (C) 50 (D) 60 (E) 84

19. There are ten tokens, 3 black and 7 white, are to be randomly placed into each of the unit squares of a 2×5 board, where each unit square can only fit exactly one token. How many ways are there to place the tokens such that no two black tokens are adjacent to each other?
(A) 30 (B) 36 (C) 38 (D) 40 (E) 60

20. The four circles below have some number of points on their circumference. Connect all the points on the same circle using straight lines and count the number of regions these segments have partitioned the circle into.



2 points, 2 regions. 3 points, 4 regions. 4 points, 8 regions. 5 points, 16 regions.

If there are 6 points on the circumference of a circle, how many regions have been partitioned at the most?
(A) 24 (B) 30 (C) 31 (D) 32 (E) 40

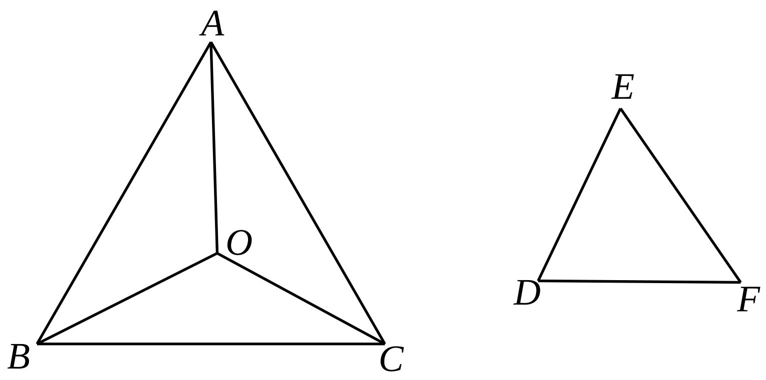
Questions 21-25, 6 marks each

21. Given $x\neq n+0.5$, where n is an integer.
Let $[x]$ represent the integer closest to x. For example, $[2.4]=2$ and $[2.6]=3$.
What is the numerical value of $[\sqrt{1\times2}]+[\sqrt{2\times3}]+[\sqrt{3\times4}]+...+[\sqrt{40\times41}]$?

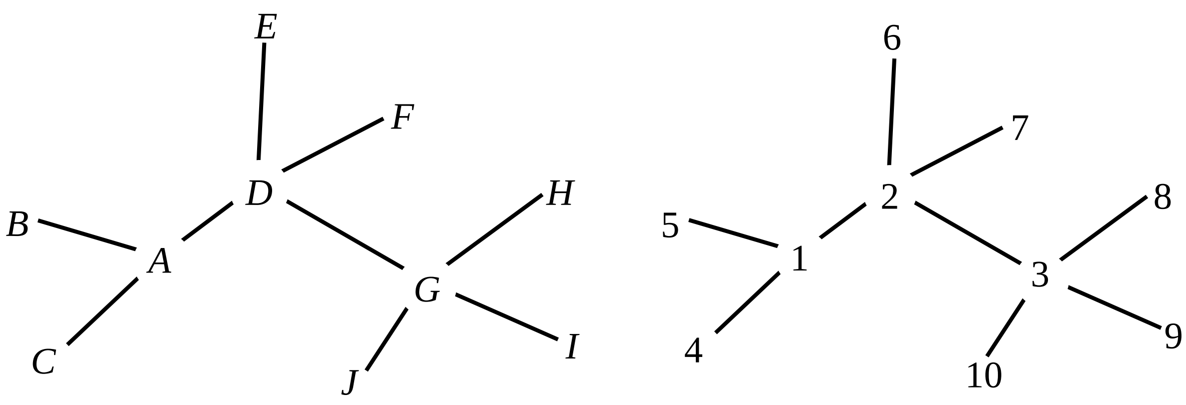
22. A party was attended by some number of families, where each family consists of a couple and their kid(s) and it is known that a family can have at most 10 kids. It is known that one dad, one mom and one kid, all from different families, will be selected to play in a game, then there are 4884 different ways to choose. How many kids attended the party?

23. Two arithmetic progressions, where each corresponding term of both sequences are multiplied to each other and we obtain the following sequence: 1, 10, 100, …. Find the 6th term of the resulting sequence.

24. Let O be a point inside equilateral triangle ABC such that $\angle AOB = 115°$ and $\angle BOC = 125°$. If DEF is a triangle such that $EF = OA$, $FD = OB$ and $DE = OC$, what is the maximum angle measure, in degrees, of triangle DEF?



25. The diagram below on the left shows a map, where cities and roads are interconnected, such that there is unique path, between any two cities, without passing any road more than once. Let x be the number of ways to labell the cities by the numbers 1, 2, …, 10 such that every path starting from city A are all strictly increasing. What is the value of $\frac{x}{20}$? The diagram below on the right shows one example.



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