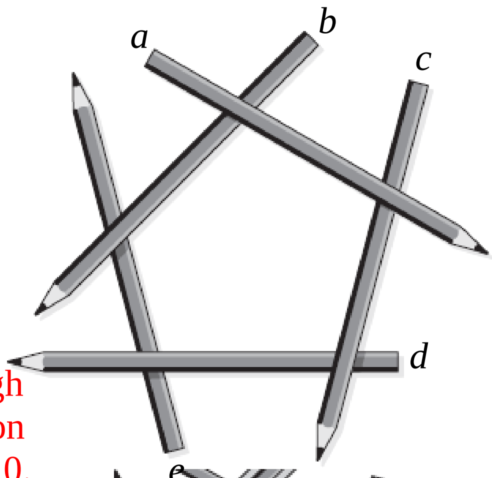
Solution to
2021~2022 International Mathematics Assessment for Schools
Round 1 of Middle Primary Division

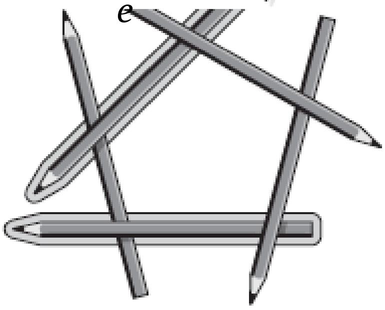
1. Five identical pencils are positioned on the table as shown on the right. Which two pencils are lying on the same plane?



（A）*b* and *c*　（B）*b* and *d*　（C）*b* and *e*
（D）*c* and *d*　（E）*c* and *e*

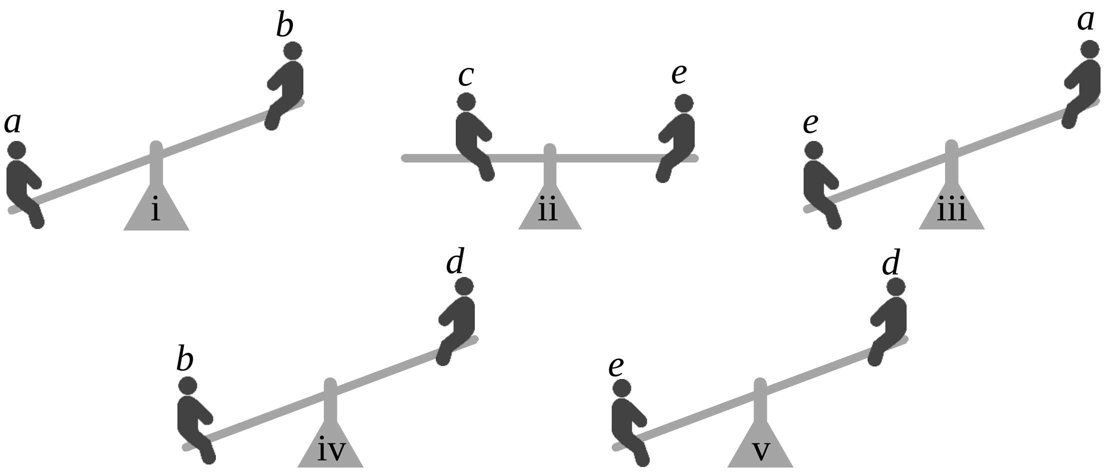
【Suggested Solution】

We denote the face of table as Level 0 and a pencil high as Level 1. Then, the nib and top of pencil *a* are both on Level 1. The nib and top of pencil *e* are both on Level 0. The nib of pencil *b* and *d* are both on Level 1 while the top of pencil *b* and *d* are both on Level 0, hence they lying in the same plane. Even the top of pencil *c* is on Level 0 but its nib is little higher than Level 1, it is not lying in the same plane with pencil *b* and *d*.



Answer：（B）

2. Five children ride on a seesaw. The diagrams below are images showing the weight comparisons between two different persons.



Who has the heaviest weight?

（A）*a*　（B）*b*　（C）*c*　（D）*d*　（E）*e*

【Suggested Solution】

From seesaw (i), we have *a* is heavier than *b*. From seesaw (ii), we have *c* is heavier than *e* . From seesaw (iii), we have *e* is heavier than *a* . From seesaw (iv), we have *b* is heavier than *d* . From seesaw (v), we have *e* is heavier than *d*. So the weights from heaviest to lightest is *c*, *e*, *a*, *b* and *d*.

Answer：（C）

3. Tom and Tim play a badminton match. The first player to reach 21 points wins a game and the first player to win two games wins the match. If Tom wins the first and third games and gets a total of 60 points for the whole match, what score did Tom get in the second game?

(A) 8 (B) 10 (C) 15 (D) 18 (E) 20

【Suggested Solution】

Since Tom wins the first and third games, he gets 21 points in each of the first and third game. So Tom gets $60-21-21=18$ points in the second game.

Answer：(D)

4. Which of the following is closest to 2021 kg?

(A) 2 cars, and the weight of each car is 1013 kg.

(B) 21 motorcycles, and the weight of each motorcycle is 101 kg.

(C) 155 bicycles, and the weight of each bicycle is 13 kg.

(D) 61 desks, and the weight of each desk is 33 kg.

(E) 44 refrigerators, and the weight of each refrigerator is 46 kg.

【Suggested Solution】

The total weight of 2 cars is $2\times1013=2026$ kg,
the total weight of 21 motorcycles is $21\times101=2121$ kg,
the total weight of 155 bicycles is $155\times13=2015$ kg.
the total weight of 61 desks is $61\times33=2013$ kg,
the total weight of 44 refrigerators is $44\times46=2024$ kg,
Since $2026-2021=5$ 、 $2121-2021=100$ 、 $2021-2015=6$ 、 $2021-2013=8$ 、 $2024-2021=3$ 。So 44 refrigerators are closest to 2021 kg.

Answer：(E)

5. Let ★ be a positive integer such that $(★-2)\times(★+2)=2021$, what is the value of ★?

(A) 42 (B) 43 (C) 44 (D) 45 (E) 47

【Suggested Solution 1】

Since we know that $2021=43\times47$, then it easy to see that $★=45$.

【Suggested Solution 2】

Observe that the equation is in the form $(★-2)\times(★+2)=★\times★-2\times2=2021$, simplifying we get $★\times★=2021+4$, which yields $★\times★=2025$. Since ★ is positive integer, we have $★=45$.

Answer：(D)

6. What is the value of $2021-2020+2019-2018+2017+...-2+1$?

(A) 1 (B) 1010 (C) 1011 (D) 1012 (E) 2021

【Suggested Solution】

$$2021-2020+2019-2018+2017+...-2+1$$
$$=(2021-2020)+(2019-2018)+(2017-2016)+...+(3-2)+1$$
$$=1011$$

Answer：(C)

7. Josh has 3 dogs, namely Jack, Sparrow and Tom, where each of them have different weights. If Jack and Sparrow weigh 12 kg together, Sparrow and Tom weigh 16 kg together and Tom and Jack weigh 14 kg together, then what is the total weight, in kg, of all 3 dogs?

(A) 21kg (B) 24 kg (C) 26 kg (D) 28 kg (E) 42 kg

【Suggested Solution】

If we add the weight of each pair then each dog participates 2 times in the sum. Thus the total weight of all 3 dogs is $\frac{12+14+16}{2}=21$ kg.

Answer：(A)

8. A certain year in the 21st Century is a perfect square number. What year is it? (Note: $1=1^2$, $4=2^2$, $9=3^2$, …, so we call 1, 4, 9, … perfect square numbers.)

(A) 1936 (B) 2021 (C) 2025 (D) 2116 (E) 2209

【Suggested Solution】

All the years of the 21st century are referred as four-digit numbers such as 20□□, that is, a total of 100 years from 2000 to 2099. Because $44^2=1936$ and $46^2=2116$ so that $44^2<20□□<46^2$, but we know that $45^2=2025$. It follows the required year number is 2025. Hence, the year number is 2025.

Answer：(C)

9. Peter keeps all of his socks in a messy non-transparent drawer under his bed. He has 8 black socks and 6 white socks. He takes the socks out from the box, one sock at a time. How many attempts does he need to make sure that he gets 2 socks of the same colour? (Note: There is no difference between a left and a right sock.)

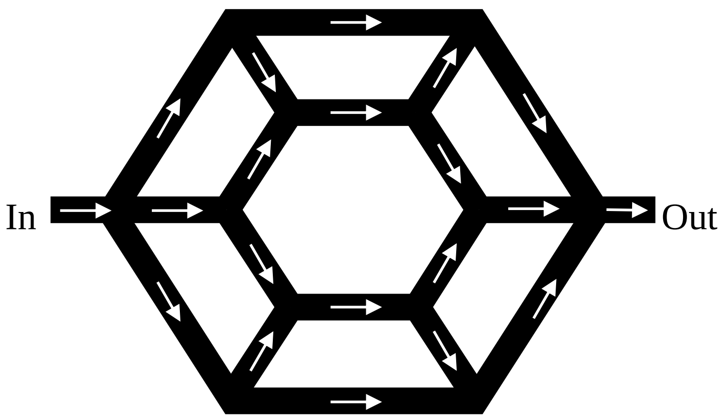
(A) 2 (B) 3 (C) 4 (D) 7 (E) 9

【Suggested Solution】

He needs 3 attempts, works case scenario is he takes a 2 socks of different colour in the first 2 tries and the 3rd one will be the same colour as one of the first 2.

Answer：(B)

10. The puzzle shown in the diagram below has one rule: Always follow the direction of the arrows. How many allowable routes from “in” to “out” adhere to the rule?

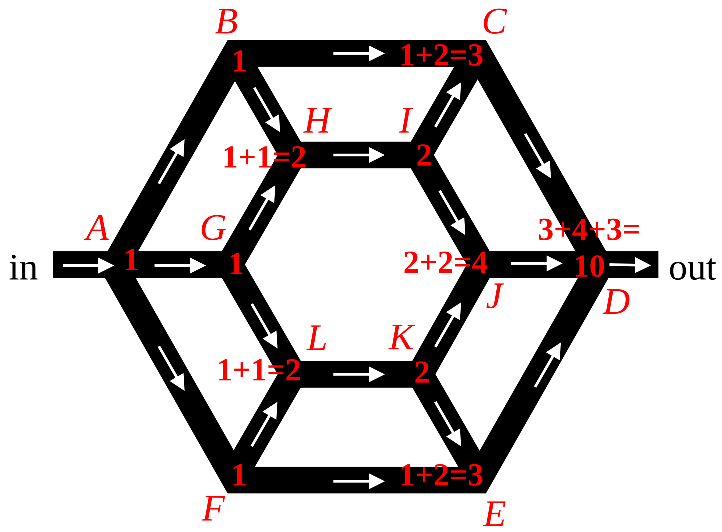


(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

【Suggested Solution】

The are 10 allowable routes.

The number of routes to each intersection is shown in the diagram below.



From A, there is only a single path to each of B, G and F. To reach H, one can either pass through B or G, and so there are $1+1=2$ ways. To reach I, one can only come from H and hence, there are 2 ways to reach I. Meanwhile, to reach C, one can come from B or I, hence, a total of $1+2=3$ ways. Using the same reasoning, we obtain the following conclusions:

To reach intersection L, there are 2 ways.

To reach the intersection K, there are 2 ways.

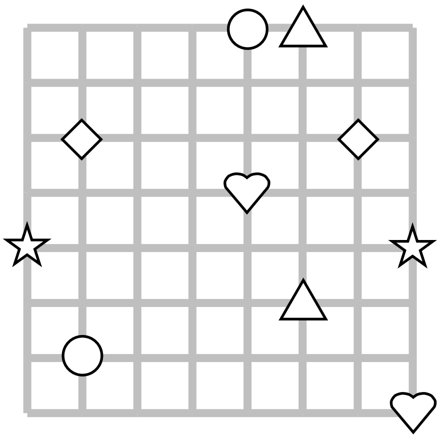
To reach the intersection E, either one comes from F or K, hence a total of $1+2=3$ ways.

To reach the intersection J, either one comes from I or K, hence a total of $2+2=4$ ways.

To reach the intersection D, one can come from C, J or E, and so we conclude that there is a total of $3+4+3=10$ different paths.

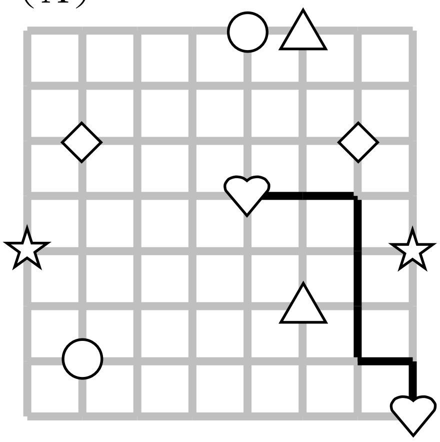
Answer：(E)

11. The diagram below contains five pairs of figures (square, triangle, circle, heart and star) that are placed on a grid. We must connect any two same figures using a single path such that all connecting paths must run only along the grey segments of the grid and no connecting paths may intersect.

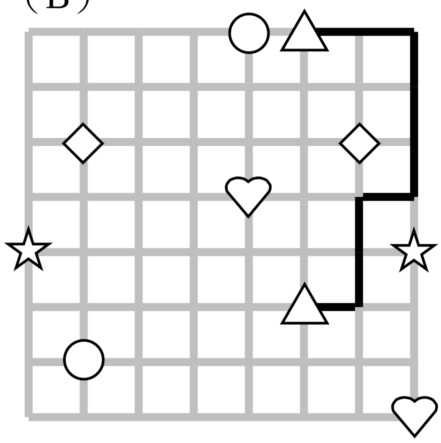


Jane draws five set of connecting paths, as shown in the options. Exactly four of them are satisfying the conditions. Which option is **NOT** a correct set of connecting paths?

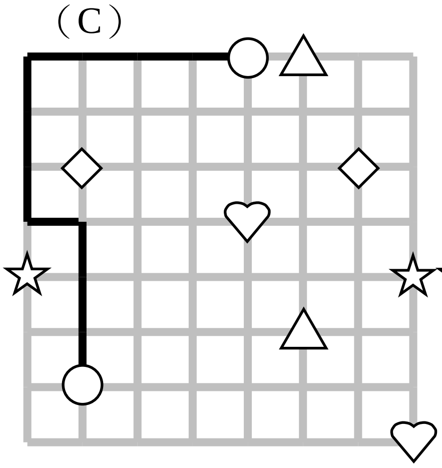
（A）



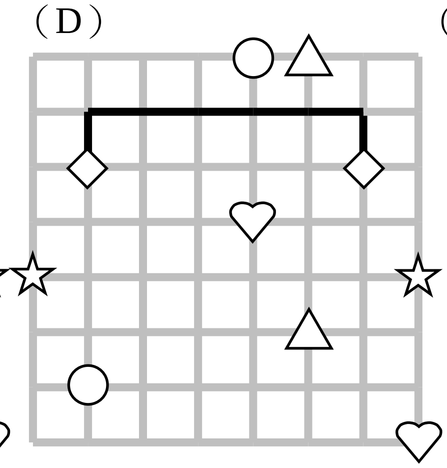
（B）



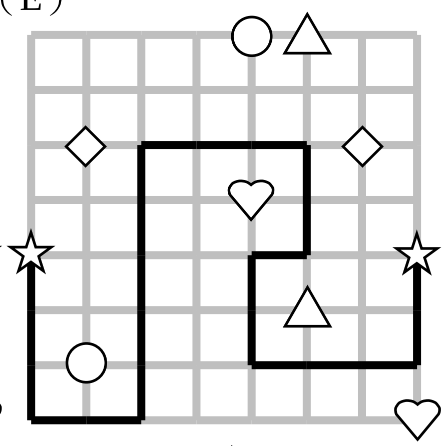
（C）



（D）



（E）

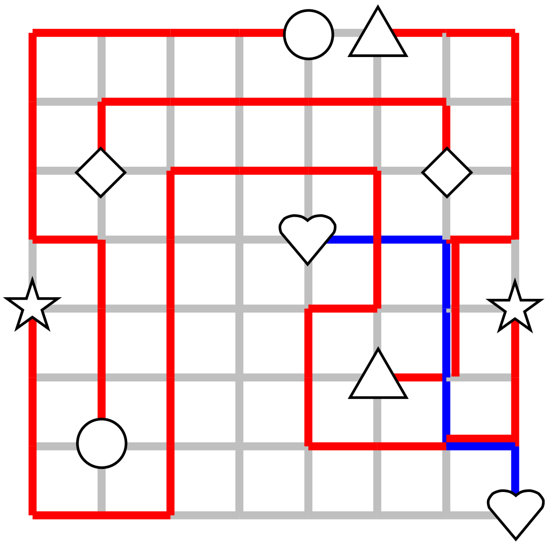


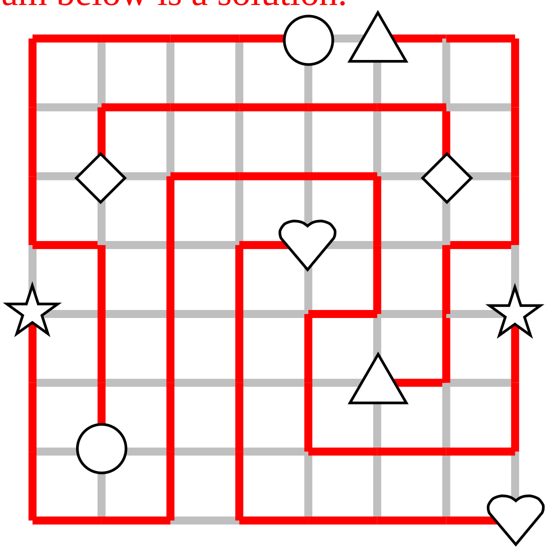
【Suggested Solution】

As we combine the five connecting paths, we can find that the path in option (A) will intersect with the paths in option (B) and (E).

Since exactly four paths are satisfying the conditions, so the path in option (A) is not correct.

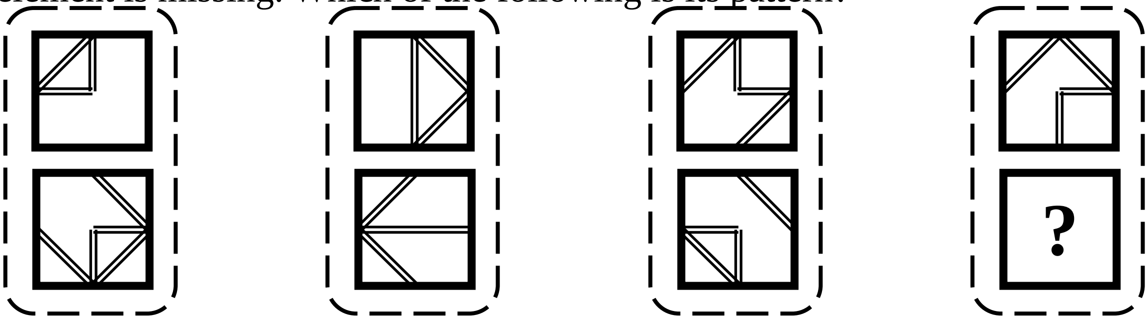
The diagram below is a solution.

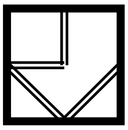


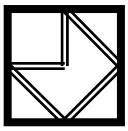


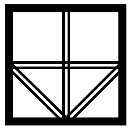
Answer：（A）

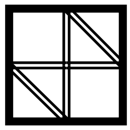
12. There are four pairs of square patterns as shown below. Inside each square, there is a wire element. In one of the squares (marked with a question mark), the wire element is missing. Which of the following is its pattern?

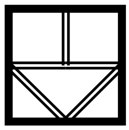


(A) 

(B) 

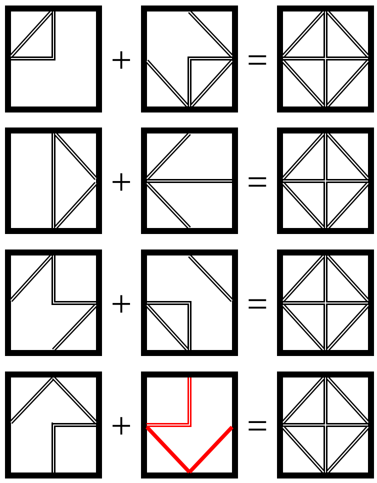
(C) 

(D) 

(E) 

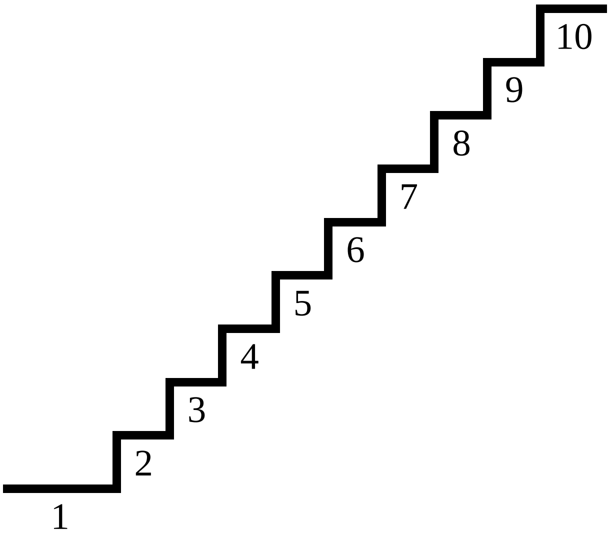
【Suggested Solution】

Each pair of patterns always gives the same pattern; see the diagrams below. The missing wire element is highlighted.



Answer：(A)

13. Alan and Benjie use the staircase shown in the diagram below to play a game where the goal is to reach a certain level on the staircase first. At the beginning, both of them will start in step 1, and in each turn, they will be playing the “Rock, Paper and Scissors” game to move around the staircase. The winner in each game gets to move 4 consecutive steps upward (or downward or a combination of both) for showing a rock; 5 consecutive steps for showing scissors and 6 consecutive steps for showing a paper. So, for example, when someone reaches step 10, he must go down to the step 9 and so on, finally returning to the step 1 and then goes back up doing the same procedure again until somebody wins. To illustrate further, when a person shows a rock on the first game and wins and then in the second game shows a paper and wins, he will then move and land on step 9.



Since Alan is currently on step 1 of the stairs, what is the least number of times he has to win in order to reach step 2? Note: If a game is a tie (where both players show the same hand, i.e. paper and paper), then they don't move.

（A）2 （B）3 （C）4 （D）5 （E）8

【Suggested Solution】

Let us find the possible combination of the game, in order to answer the problem correctly.

Since Alan stopped on the level 2. If the answer just depends on one game, then it is impossible. Therefore, it must be from the level 10 moving down to the level 2. There are 9 move-up steps and 8 move-down steps, with a total of 17 steps. Now, let us use the sum of 4 steps, 5 steps and 6 steps to express 17. So, we have $4+4+4+5=17$ or $6+6+5=17$, it is either one of the two, consider Alan win at least 3 times, which will be Alan won 1 time with scissors and 2 times with paper.

Answer：（B）

14. A young-looking mother took her child to the park to play. People thought she was the child's elder sister. Curiously, one person asked her age and her smart child replied: "Four years ago, my mother was 7 times my age, but now she is 4 times my age." How old is the mother?

（A）21 （B）24 （C）28 （D）32 （E）36

【Suggested Solution 1】

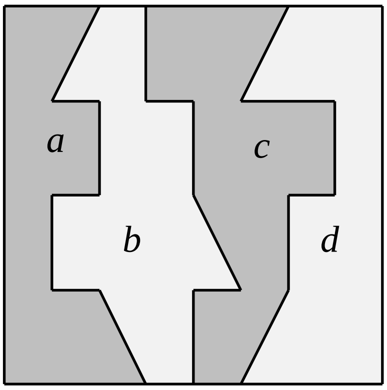
Since the age difference of mother and her child are never change, hence four years ago the difference was $7-1=6$ times of the child's age, and now the difference was $4-1=3$ times of the child's age. After four years, the difference from 6 times change to 3 times of the child's age, then we can get of the child's age was double. Hence, the child is 8 years old now. Therefore, the mother's age is 32 years old now.

【Suggested Solution 2】

Suppose the ages of the mother and child at present are A and B respectively, and two equations can be listed by the given conditions as follows: $(B-4)\times 7=A-4$ and $A=4B$. Therefore, the child's age is $B=8$ and the mother's age is $A=8\times 4=32$ years old.

Answer：（D）

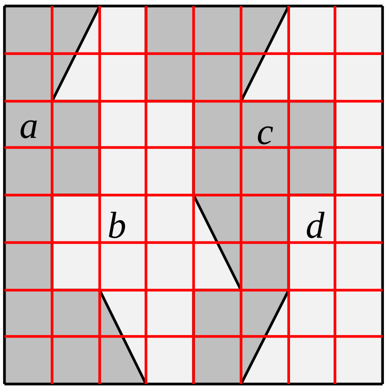
15. The diagram below is a square that is divided into four parts. Which two of them have the same area?



（A） *a* and *b* （B） *a* and *c* （C） *b* and *c* （D） *b* and *d* （E） *c* and *d*

【Suggested Solution】

Add the grid as the diagram shown on the right.
Thus the area of part *a* is 14 unit square, the area of part *b* is 17 unit square, the area of part *c* is 17 unit square, the area of part *d* is 16 unit square. Hence part *b* and *c* have the same area.



Answer：(C)

16. Select two numbers from 2, 3, 4, 5, 6, 7 and 8 to form a simplified proper fraction. If the product of any two such simplified proper fractions is $\frac{1}{2}$, they are considered as one pair. How many pairs are there in total?

（A） 4 （B） 5 （C） 6 （D） 7 （E） 8

【Suggested Solution】

Let us begin with the smallest number and be sure the fractions that you formed cannot be reduced.

When the denominator is 3, we have $\frac{2}{3}$.

When the denominator is 4, we have $\frac{3}{4}$.

When the denominator is 5, there are 3 simplified fractions can be formed: $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$.

When the denominator is 6, we have $\frac{5}{6}$.

When the denominator is 7, there are 5 simplified fractions can be formed: $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$, $\frac{6}{7}$

When the denominator is 8, there are 3 simplified fractions can be formed: $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$.

The product of any of the two simplified fractions as mentioned above will be $\frac{1}{2}$, they are:

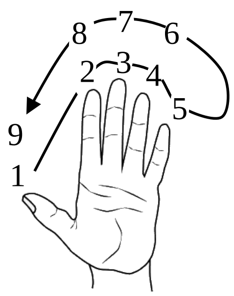
First Pair: $\frac{2}{3}\times\frac{3}{4}=\frac{2}{4}=\frac{1}{2}$; Second Pair: $\frac{3}{5}\times\frac{5}{6}=\frac{3}{6}=\frac{1}{2}$;

Third Pair: $\frac{5}{8}\times\frac{4}{5}=\frac{4}{8}=\frac{1}{2}$; Fourth Pair: $\frac{7}{8}\times\frac{4}{7}=\frac{4}{8}=\frac{1}{2}$

Therefore, there are four possible pairs in total.

Answer：(A)

17. Let's play a game where we start counting from the thumb with the number 1, the index finger is 2, the middle finger is 3, the ring finger is 4 and the little finger is 5, then after which we count in backward order, where the ring finger is 6, the middle finger is 7, the index finger is 8, and the thumb is 9; then we count in forward order again, and we keep on counting in this manner, as shown on the diagram. Which finger will it land into when I count to 2021?



(A) thumb (B) index (C) middle (D) ring (E) little

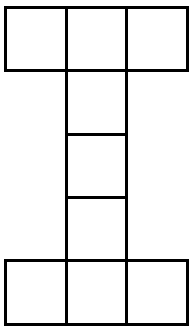
【Suggested Solution】

Obviously, solving this problem is not relying on counting from finger to finger, but finding out the pattern. From the given diagram, we know that 8 numbers form a cycle, so no matter how big the number is, we only need to divide by 8 first, and after knowing the remainder and starting counting the remainder, then we will know the number is located in which finger.

Divide 2021 by 8, and the final remainder is 5. Just simply count the final count to the little finger.

Answer：(E)

18. The diagram on the right is made up of nine identical squares which form the letter "I". How many different rectangles located in different positions (including squares) are there in the diagram?



(A) 9 (B) 12 (C) 18 (D) 22 (E) 25

【Suggested Solution】

The only possible rectangles we can have in the diagram are as follows: 1 by 1 square, 1 by 2 rectangle, 1 by 3 rectangle, 1 by 4 rectangle and 1 by 5 rectangles.

Counting them, we have nine 1 by 1 squares, eight 1 by 2 rectangles, five 1 by 3 rectangles, two 1 by 4 rectangles and one 1 by 5 rectangle.

In total, there are $9+8+5+2+1=25$ rectangles.

Answer：(E)

19. Four persons *A*, *B*, *C* and *D* will participate in a 100-meter race. The following table shows the predicted rankings of each of these four participants before the race. Line ① is participant *B*'s prediction. After the race, all four of them checked the final ranking and after comparing it with their predictions, they found out that nobody predicted their own personal ranking correctly. It is known that everyone has correctly predicted the winner of at least one from the first to third placers, none of them correctly predicted who finished last and exactly two participants correctly predicted the third place winner. Who are the participants that predicted line ②, line ③ and line ④? Note: write the answer in this order.

	First Place	Second Place	Third Place	Four Place
①	D	B	C	A
②	D	A	B	C
③	A	B	C	D
④	C	A	B	D

（A） C, D, A （B） C, A, D （C） D, C, A （D） D, A, C （E） A, C, D

【Suggested Solution】

This is like a guessing game. We must use reasoning skill to give the correct ranking. First, since no one guessed the fourth placer correctly, therefore, the participant B was the fourth placer. And we know that line ② and line ④, guess that participant B won the third placer, which is wrong, so the third placer must be participant C, line ① and line ③, guess that participant B won the second placer, which is wrong, so the second placer must be participant A and this implies *D* is the first placer.

Hence, the first placer is participant D, second placer is participant A, third placer is participant C and fourth placer is participant B.

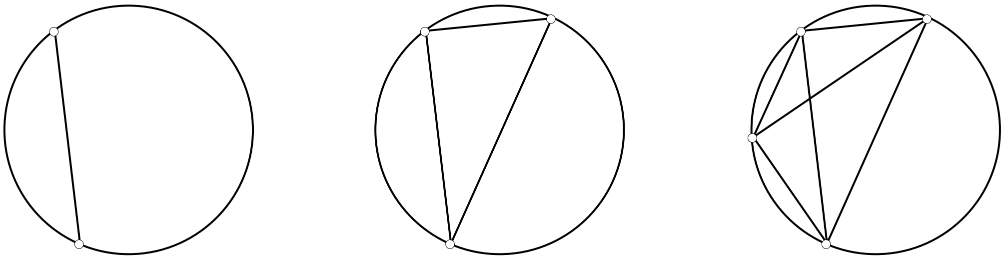
Then we will find out who are those people who do the prediction of line ②, line ③ and line ④? Let's look at the prediction of line ②, since the ranking of participant B and participant C are wrong, because participant B is the who do the prediction of line ①, so line ② should be the prediction of participant C. Carefully observe the prediction of line ④, only the ranking of participant A is correct, so participant A cannot be the one who predict line ④, because no one has predict his own ranking.

Thus, in summary, we have line ① is predicted by participant B, line ② is predicted by participant C, line ③ must be predicted by participant A and line ④ is predicted by participant D .

	First Place	Second Place	Third Place	Four Place
Final rankings	D	A	C	B
① B	D	B	C	A
② C	D	A	B	C
③ A	A	B	C	D
④ D	C	A	B	D

Answer：(B)

20. The three circles below have some number of points on their circumference. Connect all the points on the same circle using straight lines and count the number of regions these segments have partitioned the circle into.



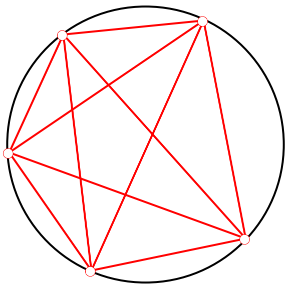
2 points, 2 regions. 3 points, 4 regions. 4 points, 8 regions.

If there are 5 points on the circumference of a circle, how many regions have been partitioned at the most?

（A）10 （B）12 （C）15 （D）16 （E）20

【Suggested Solution】

Let any three segments not meet at a same point then we can get at most 16 regions as the diagram shown.



5 points,
16 regions.

Answer：(D)

21. In a race, each athlete must complete a 100-meter distance at their own fixed speed, and the athlete who reaches the finish line first wins. If athlete A has reached the finish line, athlete B still has 10 meters to complete the race; and when athlete B reaches the finish line, athlete C is still 20 meters away. How many meters was athlete A ahead of athlete C?

【Suggested Solution】

Since each athlete finished the race at his own fixed speed. When athlete A reached the finish line, athlete B was still 10 meters far, and the ratio of the distance of athlete A to athlete B in the same time was $\frac{100}{90}$; that is, athlete B's speed was 90% of athlete A's speed. Similarly, athlete C's speed was 80% of athlete B's speed. It follows athlete C's speed was $90\% \times 80\% = 72\%$ of athlete A's speed.

Hence, when athlete A reaches the finish line, athlete C should be at 72 meters. Thus, athlete A is $100 - 72 = 28$ meters ahead of athlete C.

Answer：028

22. There are four cards with digits 0, 1, 2 and 3 written on them. Choose any three cards and arrange them to form all possible three-digit numbers (where 0 is not allowed to be the leading digit). What is the average of all possible three-digit numbers that are odd?

【Suggested Solution】

From the given information, there are only two even digits and two odd digits, we have 2 way choose the unit digit, then we have 2 way choose the leading digit, we have 2 way to arrange the tens digits. Hence we have $2\times2\times2=8$ such 3-digit numbers, we list down all of those 3-digit numbers:

103, 123, 201, 203, 213, 231, 301, 321.

their sum is 1696 and its average is $1696\div8=212$.

Answer：212

23. Fill-in the numbers 1, 2, 3, 4, 6, 9, 12, 18, and 36 into each of the unit squares exactly once on the grid below, so that the product of the three numbers on each horizontal, vertical and diagonal lines are all equal. Which number must be filled in the unit square marked with "☆"?

	☆	

【Suggested Solution 1】

The product of those nine numbers is $1\times2\times3\times4\times6\times9\times12\times18\times36=$
$1\times2\times3\times2\times2\times2\times3\times3\times3\times2\times2\times3\times2\times3\times3\times2\times2\times3\times3$

Hence the product of each horizontal, vertical and diagonal line all equal to $2\times2\times2\times3\times3\times3=216$. We have

$$216=36\times6\times1=36\times3\times2=18\times12\times1=18\times6\times2$$
$$=18\times4\times3=12\times9\times2=12\times6\times3=9\times6\times4$$

Since 6 appear in four multiplication, it must placed at ☆.

We can complete the table as shown on the right.

12	1	18
9	6	4
2	36	3

【Suggested Solution 2】

Labelled the unit squares as diagram shown. Since the product of the three numbers on each horizontal, vertical and diagonal lines are all equal we have $a\times h=b\times g=c\times f=d\times e$ 。Then we can get :
$1\times36=2\times18=3\times12=4\times9$.

a	b	c
d	☆	e
f	g	h

Hence, 6 must placed at ☆. We can complete the table as shown below.

12	1	18
9	6	4
2	36	3

Answer：006

24. There are three kinds of tokens having different weights: 5 grams, 10 grams and 20 grams. The total weight of 19 tokens is 250 grams. If the number of 5 gram tokens and the number of 20 gram tokens are interchanged, then the total weight of tokens are reduced to 190 grams. What is the number of tokens having a weight of 10 grams?

【Suggested Solution】

Suppose originally, the number of tokens of the 20 grams is 1 more than the number of tokens of 5 grams, now, let us interchange the number of tokens of the 20 grams and the number of tokens of 5 grams, then the total tokens is $20-5=15$ grams less

than the original number.
From the given information, we know the original weight of those 19 tokens is 250 and after exchanging the number of tokens between the 5 grams and 20 grams, there are $250-190=60$ grams less. Because $60\div15=4$, this indicates that there are 4 more tokens of 20 grams than 5 grams.
When reduced 4 pieces of tokens of 20 grams then the number of tokens of 20 grams than 5 grams is the same and the total tokens is $250-20\times4=170$ grams.
At this time, the number of tokens becomes $19-4=15$ pieces.
If 15 tokens are all of 10 grams, which has a total of 150 grams. But there must be a total tokens of 170 grams, it is 20 grams less, so there are tokens of 5 grams and 20 grams. If we merged one piece of 5 grams token and one piece 20 grams token as a big token, then 1 big token is $(5+20)-10\times2=5$ more than two tokens of 10 grams. It follows the number of big tokens are $20\div5=4$ pieces and this implies there are $(170-25\times4)\div10=7$ pieces of tokens of 10 grams. Therefore, there are 4 tokens of 5 grams, 8 tokens of 20 grams and 7 tokens of 10 grams.

Answer：007

25. A strange math teacher gave his students this problem. He first wrote the following eight digits: 1, 2, 3, 4, 5, 7, 8 and 9 on the board. He then asked his students to split these digits into two groups, such that each group has four digits; and then arrange and combine the four digits in each group to form two 2-digit numbers and add them together. Finally, the result of adding the two 2-digit numbers in each group must be the same 2-digit number. What the largest possible value of this sum?

【Suggested Solution 1】

As the student of this strange math teacher, he or she must split the 8 digits into two groups then combine these 4 digits in each group into two 2-digit numbers and add them together. Since the result of adding the two numbers in each group is a same 2-digit number, so the first digit of these two number have at most one of digits 5, 7, 8, 9 otherwise their sum will became 3-digit number. We can not choose 9 as one of first digit or their sum will carry and became 3-digit number.
If we choose 7 and 8 for each one number's first digit of these two group, the sum of these two group must be $\overline{7w}+\overline{2x}=\overline{8y}+\overline{1z}$, where *w*, *x*, *y*, *z* be one of digits 3, 4, 5,9.
There must have a carry on, so it is impossible.
If we choose 5 and 8, the sum of these two group have two case $\overline{5w}+\overline{4x}=\overline{8y}+\overline{1z}$, where *w*, *x*, *y*, *z* be one of digits 2, 3, 7, 9, or the sum of these two group must be $\overline{5w}+\overline{3x}=\overline{8y}+\overline{1z}$, where *w*, *x*, *y*, *z* be one of digits 2, 4, 7, 9. The former must have a curry on, so it is impossible. For the latter, we may let (*w*, *x*) = (7, 9) or (9, 7) and (*y*, *z*) = (2, 4) or (4, 2) and obtained their same sum both are 96.
If we choose 5 and 7, the sum of these two group have two case $\overline{5w}+\overline{4x}=\overline{7y}+\overline{2z}$, where *w*, *x*, *y*, *z* be one of digits 1, 3, 8, 9, or the sum of these two group be $\overline{5w}+\overline{3x}=\overline{7y}+\overline{2z}$, where *w*, *x*, *y*, *z* be one of digits 1, 4, 8, 9, or the sum of these two group must be $\overline{5w}+\overline{2x}=\overline{7y}+\overline{1z}$, where *w*, *x*, *y*, *z* be one of digits 3, 4, 8, 9. The

former two case must have a curry on, so it is impossible. For the last, we may let (w, x) = (8, 9) or (9,8) and (y, z) = (3, 4) or (4,3) and obtained their same sum both are 87. Hence, the largest value of this sum is 96.

【Suggested Solution 2】

Since total sum of all digits is 39 have remainder 3 when divided by 9. Hence, the resulting number should have remainder 6 divided by 9. The largest such 2-digit number 96. And 96 can be realized as $57+39$ and $78+21$, so the largest is 96.

Answer：096